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R-parity violation and non-abelian discrete family symmetry

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Abstract: We investigate the implications of R-parity violating operators in a model with family symmetry. The family symmetry can determine the form of R-parity violating operators as well as the Yukawa matrices responsible for fermion masses and mixings. In this paper we consider a concrete model with non-abelian discrete symmetry Q_6 which contains only three R-parity violating operators. We find that ratios of decay rates of the lepton flavor violating processes are fixed thanks to the family symmetry, predicting $BR(\tau \to 3e)/BR(\tau \to 3\mu) \sim 4m_\mu^2/m_\tau^2$.

Keywords: Supersymmetry Phenomenology, Discrete and Finite Symmetries, Supersymmetric Standard Model.

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Contents

1. Introduction

Despite the remarkable success of the gauge sector of the Standard Model (SM), there still exist some problems in the Higgs and Yukawa sectors. The Yukawa matrices are responsible for the masses and mixings of matter fermions: quarks and leptons. The Yukawa sector in the SM can give experimentally consistent masses and mixings, because it contains more free parameters than the number of observables in general. There is no predictivity in the Yukawa sector because of this redundancy of the parameters. One of the ideas to overcome this issue is to introduce a family symmetry (flavor symmetry), which is the symmetry between generations. In this paper we consider a concrete model which is symmetric under the binary dihedral group Q_6 [1, 2].

On the other hand, in the Higgs sector, the most important problem is that the Higgs boson has not been experimentally discovered yet. Discovery of the Higgs boson is expected at the Large Hadron Collider (LHC). In the SM, the Higgs mass is quadratic divergent. This problem is solved by introducing Supersymmetry (SUSY) at $O(1)$ TeV. The Minimal Supersymmetric Standard Model (MSSM) has low energy SUSY. In general it contains gauge symmetric, lepton and baryon number violating operators

$$W_{\not R} = \frac{1}{2}\lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2}\lambda''_{ijk} U^c_i D^c_j D^c_k + \mu_i H_u L_i \tag{1.1}$$

in addition to the usual Yukawa couplings and μ-term. The asymmetric properties $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$ mean that $9+27+9+3 = 48$ (complex) parameters are included in these interactions. These couplings generate unacceptable processes such as Lepton Flavor Violating (LFV) processes and proton decay. The conservation of R-parity [11, 12]

$$R = (-1)^{3B+L+2s}, \tag{1.2}$$

where B, L and s denote baryon, lepton number and spin of the particles, respectively, is one possibility to forbid the couplings eq. (1.1). From the definition, R-parity is $+1$ for all SM particles and -1 for all their superpartners. However, R-parity is not the only possible choice to forbid the interactions. Matter- or lepton- and baryon-parity [13] can be also a possibility. On the other hand, without R-parity, these coupling constants have to be strictly constrained not to conflict with experimental data. Constraints on the R-parity violating couplings have been obtained by many authors from LFV processes [14 − 19], neutrino mass [14, 20 − 23], neutral meson system [24 − 32], proton decay [16, 33, 34], and so on [12, 35, 38].

Family symmetries also constrain the form of R-parity interactions [22, 33, 36, 37] as well as the Yukawa matrices. In the model that we consider [1, 2], the Q_6 family symmetry reduces the 45 trilinear couplings to three: λ, λ'_1 and λ'_2. The baryon number violating couplings $\lambda''_{ijk}U^c_i D^c_j D^c_k$ are forbidden by the symmetry in our model, so it is guaranteed by the symmetry that the R-parity violating operators do not induce proton decay.

In this paper, we study the phenomenology of the three R-parity violating interactions in the model with Q_6 family symmetry [1, 2]. First, we obtain upper bounds on three coupling constants λ and $\lambda'_{1,2}$ from the experimental constraints. Next we focus on LFV processes induced by λ. The λLLE^c coupling generates the LFV decays $\ell^-_m \to \ell^-_i \ell^-_j \ell^+_k (m, i, j, k$ denote the flavor of the charged lepton) at tree level, and the Branching Ratios (BR) of the decay processes are proportional to λ^4. Therefore, the ratios of these processes are independent of λ and can be predicted unambiguously to be $BR(\tau \to eee)/BR(\tau \to \mu\mu\mu) \sim 4m^2_\mu/m^2_\tau$. It reflects the properties of the family symmetry. We introduce the Q_6 symmetric model in the next section, and derive the predictions in the section 3.

2. The model

2.1 Group theory of Q_6

The binary dihedral group $Q_N(N = 2, 4, 6, \ldots)$ is a finite subgroup of SU(2) and defined by the following set of $2N$ elements

$$Q_N = \left\{1, A, A^2, \ldots, A^{N-1}, B, AB, \ldots, A^{N-1}B\right\}, \tag{2.1}$$

where two dimensional representation of matrix A and B is given by

$$A = \begin{pmatrix} \cos\phi_N & \sin\phi_N \\ -\sin\phi_N & \cos\phi_N \end{pmatrix}, \; \phi_N = \frac{2\pi}{N}, \quad B = \begin{pmatrix} i & \\ & -i \end{pmatrix}. \tag{2.2}$$

Since we consider a supersymmetric model with Q_6 family symmetry, we show the multiplication rules only for the case of $N = 6$.

Q_6 group contains 2 two-dimensional irreducible representations (irreps), $\mathbf{2}_1, \mathbf{2}_2$ and 4 one-dimensional ones $\mathbf{1}_{+,0}, \mathbf{1}_{+,2}, \mathbf{1}_{-,1}, \mathbf{1}_{-,3}$, where $\mathbf{2}_1$ is pseudo real and $\mathbf{2}_2$ is real representation. In the notation of $\mathbf{1}_{\pm,n}(n = 0, 1, 2, 3)$, $\pm$ stands for the change of sign under the transformation by matrix A, and n the factor $\exp(in\pi/2)$ by B. So $\mathbf{1}_{+,0}$ and $\mathbf{1}_{+,2}$ are real representations, while $\mathbf{1}_{-,1}$ and $\mathbf{1}_{-,3}$ are complex conjugate to each other. Their group multiplication rules are given as follows [1, 2, 5]:

$$\begin{gathered}
\mathbf{1}_{+,2} \times \mathbf{1}_{+,2} = \mathbf{1}_{+,0}, \ \mathbf{1}_{-,3} \times \mathbf{1}_{-,3} = \mathbf{1}_{+,2}, \ \mathbf{1}_{-,1} \times \mathbf{1}_{-,1} = \mathbf{1}_{+,2}, \ \mathbf{1}_{-,1} \times \mathbf{1}_{-,3} = \mathbf{1}_{+,0}, \\
\mathbf{1}_{+,2} \times \mathbf{1}_{-,1} = \mathbf{1}_{-,3}, \ \mathbf{1}_{+,2} \times \mathbf{1}_{-,3} = \mathbf{1}_{-,1}, \quad \mathbf{2}_1 \times \mathbf{1}_{+,2} = \mathbf{2}_1, \quad \mathbf{2}_1 \times \mathbf{1}_{-,3} = \mathbf{2}_2, \\
\mathbf{2}_1 \times \mathbf{1}_{-,1} = \mathbf{2}_2, \quad \mathbf{2}_2 \times \mathbf{1}_{+,2} = \mathbf{2}_2, \quad \mathbf{2}_2 \times \mathbf{1}_{-,3} = \mathbf{2}_1, \quad \mathbf{2}_2 \times \mathbf{1}_{-,1} = \mathbf{2}_1,
\end{gathered} \tag{2.3}$$

$$\begin{array}{ccccccccc}
\mathbf{2}_1 & \times & \mathbf{2}_1 & = & \mathbf{1}_{+,0} & + & \mathbf{1}_{+,2} & + & \mathbf{2}_2 \\
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \times & \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} & = & (x_1y_2 - x_2y_1) & & (x_1y_1 + x_2y_2) & & \begin{pmatrix} -x_1y_2 - x_2y_1 \\ x_1y_1 - x_2y_2 \end{pmatrix},
\end{array} \tag{2.4}$$

$$\begin{array}{ccccccccc}
\mathbf{2}_2 & \times & \mathbf{2}_2 & = & \mathbf{1}_{+,0} & + & \mathbf{1}_{+,2} & + & \mathbf{2}_2 \\
\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} & \times & \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} & = & (a_1b_1 + a_2b_2) & & (a_1b_2 - a_2b_1) & & \begin{pmatrix} -a_1b_1 + a_2b_2 \\ a_1b_2 + a_2b_1 \end{pmatrix},
\end{array} \tag{2.5}$$

$$\begin{array}{ccccccccc}
\mathbf{2}_1 & \times & \mathbf{2}_2 & = & \mathbf{1}_{-,3} & + & \mathbf{1}_{-,1} & + & \mathbf{2}_1 \\
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \times & \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} & = & (x_1a_2 + x_2a_1) & & (x_1a_1 - x_2a_2) & & \begin{pmatrix} x_1a_1 + x_2a_2 \\ x_1a_2 - x_2a_1 \end{pmatrix}.
\end{array} \tag{2.6}$$

In what follows we construct a concrete model with Q_6 family symmetry by using of the multiplication rules above.

2.2 Q_6 assignment and superpotential

We show the Q_6 assignment of the quark, lepton and Higgs chiral supermultiplets below, where Q_I, Q_3, L_I, L_3 and $H_I^u, H_3^u, H_I^d, H_3^d$ stand for $\mathrm{SU}(2)_L$ doublets supermultiplets for quarks, leptons and Higgs bosons, respectively. Similarly, $\mathrm{SU}(2)_L$ singlet supermultiplets for quarks, charged leptons and neutrinos are denoted by $U_I^c, U_3^c, D_I^c, D_3^c, E_I^c, E_3^c$ and N_I^c, N_3^c. The generation indices $I, J, \ldots = (1, 2)$ are applied to the Q_6 doublet, and $i, j, \ldots = (1, 2, 3)$ to three generations throughout the paper. Y is gauge singlet Higgs supermultiplet to give neutrino mass by seesaw mechanism.

We give the Q_6 assignment to each field as follows:

$$\begin{aligned}
\mathbf{2}_1 &: \ Q_I, \\
\mathbf{2}_2 &: \ U_I^c, \ D_I^c, \ \hat{L}_I, \ E_I^c, \ N_I^c, \ H_I^u, \ \hat{H}_I^d, \\
\mathbf{1}_{+,0} &: \ L_3, \ E_3^c, \\
\mathbf{1}_{+,2} &: \ Q_3, \ Y, \\
\mathbf{1}_{-,1} &: \ U_3^c, \ D_3^c, \ H_3^u, \ H_3^d, \\
\mathbf{1}_{-,3} &: \ N_3^c.
\end{aligned} \tag{2.7}$$

In a model without R-parity conservation, there is no distinction between lepton doublet and down type Higgs doublet, because both have the same gauge quantum numbers. We have written these fields as $\hat{L}_I, \hat{H}_I^d$, and physical lepton doublet and down type Higgs doublet will be written as linear combination of these. At first we write down the superpotential in the fields with hat ($\hat{L}_I, \hat{H}_I^d$), and after that we rewrite it in the physical fields without hat (L_I, H_I^d).

Under the field assignment above, we can write down the most general, renormalizable Q_6 invariant superpotential W (without R-parity conservation):

$$W = \hat{W}_Q + \hat{W}_L + \hat{W}_\mu + \hat{W}_{\not R}, \tag{2.8}$$

where

$$\begin{aligned} \hat{W}_Q &= W_U + \hat{W}_D, \\ W_U &= Y_a^u Q_3 U_3^c H_3^u + Y_b^u Q_I \left(\sigma^1\right)_{IJ} U_3^c H_J^u - Y_{b'}^u Q_3 U_I^c \left(i\sigma^2\right)_{IJ} H_J^u + Y_c^u Q_I \left(\sigma^1\right)_{IJ} U_J^c H_3^u, \\ \hat{W}_D &= Y_a^d Q_3 D_3^c H_3^d + \hat{Y}_b^d Q_I \left(\sigma^1\right)_{IJ} D_3^c \hat{H}_J^d - \hat{Y}_{b'}^d Q_3 D_I^c \left(i\sigma^2\right)_{IJ} \hat{H}_J^d + Y_c^d Q_I \left(\sigma^1\right)_{IJ} D_J^c H_3^d \end{aligned} \tag{2.9}$$

for the quark sector,[1] and

$$\begin{aligned} \hat{W}_L &= \hat{W}_E + \hat{W}_N, \\ \hat{W}_E &= \hat{Y}_b^e \hat{L}_I E_3^c \hat{H}_I^d + \hat{Y}_{b'}^e L_3 E_I^c \hat{H}_I^d + \hat{Y}_c^e f_{IJK} \hat{L}_I E_J^c \hat{H}_K^d, \\ \hat{W}_N &= Y_a^\nu L_3 N_3^c H_3^u + Y_{b'}^\nu L_3 N_I^c H_I^u + \hat{Y}_c^\nu f_{IJK} \hat{L}_I N_J^c H_K^u + \frac{1}{2} M_N N_I^c N_I^c + \lambda_N Y N_3^c N_3^c, \\ -f_{111} &= f_{221} = f_{122} = f_{212} = 1 \end{aligned} \tag{2.10}$$

for the lepton sector. The R-parity violating trilinear coupling $W_{\not R}$ which respects the family symmetry is given by

$$\hat{W}_{\not R} = \hat{\lambda} L_3 \hat{L}_I E_I^c + \hat{\lambda}'_1 \hat{L}_I (i\sigma^2)_{IJ} Q_3 D_J^c + \hat{\lambda}'_2 \hat{L}_I (\sigma^1)_{IJ} Q_J D_3^c. \tag{2.11}$$

The family symmetry constrains not only the form of the Yukawa sector but also that of the R-parity violating couplings. Proton decay caused by dimension-four operators $\lambda''_{ijk} U_i^c D_j^c D_k^c$ are prevented since the baryon number violating interactions are forbidden by the symmetry [2]. Therefore R-parity violating trilinear terms $\hat{W}_{\not R}$ contain only three couplings $\hat{\lambda}$ and $\hat{\lambda}'_{1,2}$. It should be compared with the MSSM case in which it contains 45 (complex) trilinear couplings. We will analyze the phenomenology of the couplings eq. (2.11) in the section 3.

In the following analysis, we assume that any couplings appearing in the superpotential eq. (2.8) are real, and CP symmetry is violated spontaneously by vacuum expectation values (VEVs) of the Higgs bosons. The μ-term W_μ which includes both R-parity conserving and violating bilinear terms is discussed in the next subsection.

[1] The superpotential for the up quark sector W_U does not have hat ($\hat{}$), because it contains neither $\hat{L}_I$ nor $\hat{H}_I^d$.

2.3 Bilinear terms

In this subsection, we discuss bilinear terms and define the physical fields L_I and H_I^d. Since the lepton and down type Higgs doublet have the same gauge quantum numbers, it is natural that both $\hat{L}_I$ and $\hat{H}_I^d$ get VEVs. We assume that the fields get complex VEVs

$$\langle H_1^u\rangle = \langle H_2^u\rangle = \frac{1}{2}v_D^u e^{i\theta^u},\ \langle H_3^u\rangle = \frac{1}{\sqrt{2}}v_3^u e^{i\theta_3^u},\ \langle H_3^d\rangle = \frac{1}{\sqrt{2}}v_3^d e^{i\theta_3^d}, \tag{2.12}$$

$$\langle \hat{H}_1^d\rangle = \langle \hat{H}_2^d\rangle = \frac{1}{2}v_d e^{i\theta^d},\quad \langle \hat{L}_1\rangle = \langle \hat{L}_2\rangle = \frac{1}{2}v_L e^{i\theta^d}, \tag{2.13}$$

where the VEVs of $\hat{H}_I^d$ and $\hat{L}_I$ have the same phase. Also, the Q_6 singlet lepton doublet L_3 does not have non-zero VEV by definition. To ensure this VEV structure, the μ-term should have the symmetry

$$H_1^u \leftrightarrow H_2^u,\ \hat{H}_1^d \leftrightarrow \hat{H}_2^d,\ \hat{L}_1 \leftrightarrow \hat{L}_2, \tag{2.14}$$

and it is written as[2]

$$\begin{aligned}\hat{W}_\mu =\ & \mu_1^d H_I^u \hat{H}_I^d + \mu_3^d H_3^u H_3^d + \mu_{13}^d (H_1^u + H_2^u) H_3^d + \mu_{31}^d H_3^u (\hat{H}_1^d + \hat{H}_2^d) + \mu_{12}^d H_I^u (\sigma^1)_{IJ} \hat{H}_J^d \\ & + \mu_1^L H_I^u \hat{L}_I + \mu_{31}^L H_3^u (\hat{L}_1 + \hat{L}_2) + \mu_{12}^L H_I^u (\sigma^1)_{IJ} \hat{L}_J. \end{aligned} \tag{2.15}$$

We define the physical Higgs fields as the fields whose VEVs break $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry. So the physical Higgs fields H_I^d and lepton doublets L_I are defined by the linear combination

$$H_I^d \equiv \frac{v_d}{v_D^d}\hat{H}_I^d + \frac{v_L}{v_D^d}\hat{L}_I, \qquad L_I \equiv \frac{v_L}{v_D^d}\hat{H}_I^d - \frac{v_d}{v_D^d}\hat{L}_I, \tag{2.16}$$

where $v_D^d = \sqrt{v_d^2 + v_L^2}$. Therefore VEVs of H_I^d are

$$\langle H_I^d \rangle = \frac{v_D^d}{2} e^{i\theta^d}, \tag{2.17}$$

which break the electroweak symmetry, while L_I do not get VEVs.

The μ-term eq. (2.15) should be rewritten in the new Higgs fields H_I^d and lepton doublet L_I as follows:

$$\begin{aligned} W_\mu =\ & \mu_1 \cos\xi_1 H_I^u H_I^d + \mu_3^d H_3^u H_3^d + \mu_{13}^d (H_1^u + H_2^u) H_3^d + \mu_{31}\cos\xi_{31} H_3^u (H_1^d + H_2^d) \\ & + \mu_{12}\cos\xi_{12} H_I^u (\sigma^1)_{IJ} H_J^d + \mu_1 \sin\xi_1 H_I^u L_I \\ & + \mu_{31}\sin\xi_{31} H_3^u (L_1 + L_2) + \mu_{12}\sin\xi_{12} H_I^u (\sigma^1)_{IJ} L_J, \end{aligned} \tag{2.18}$$

where the mixing angles of the Higgs fields and lepton doublets are defined as

$$\mu_1 \sin\xi_1 = \frac{\mu_1^d v_L - \mu_1^L v_d}{v_D^d},\ \text{etc.} \tag{2.19}$$

[2] This form of the μ-term is given by introducing additional gauge singlet Higgs fields and extra discrete symmetry [1, 2]. In the present paper, we just assume the form of eq. (2.15).

with $\mu_1 = \sqrt{(\mu_1^d)^2 + (\mu_1^L)^2}$ etc.

These mixing terms of neutralinos and neutrinos generate neutrino masses proportional to $\sin^2\xi$ at tree level [21, 22].

There are particularly strong constraints on the mixing angle $\sin\xi$ coming from neutrino masses, and it means that the μ-terms and VEVs have to be aligned, $\mu_1^d/\mu_1^L \propto v_d/v_L$.[3] In the MSSM case, a neutrino mass bound requires a strong alignment of the mixing term $W \sim \mu \sin\xi H_u L_3$,

$$\sin\xi < 3\times 10^{-6}\sqrt{1+\tan^2\beta}. \tag{2.20}$$

Therefore we simply neglect the mixing terms in the following analysis.

We can rewrite the superpotential eqs. (2.8)$\sim$(2.11) in the completely same form with the physical fields, with new coupling constants defined as

$$\begin{gathered} Y_a^d = \hat{Y}_a^d, \quad Y_b^d = \tfrac{v_d}{v_D^d}\hat{Y}_b^d - \tfrac{v_L}{v_D^d}\hat{\lambda}_2', \quad Y_{b'}^d = \tfrac{v_d}{v_D^d}\hat{Y}_{b'}^d - \tfrac{v_L}{v_D^d}\hat{\lambda}_1', \quad Y_c^d = \hat{Y}_c^d, \\ Y_b^e = -\hat{Y}_b^e, \quad Y_{b'}^e = \tfrac{v_d}{v_D^d}\hat{Y}_{b'}^e + \tfrac{v_L}{v_D^d}\hat{\lambda}, \quad Y_c^e = -\hat{Y}_c^e, \\ \lambda = \tfrac{v_L}{v_D^d}\hat{Y}_{b'}^e - \tfrac{v_d}{v_D^d}\hat{\lambda}, \quad \lambda_1' = -\tfrac{v_L}{v_D^d}\hat{Y}_{b'}^d - \tfrac{v_d}{v_D^d}\hat{\lambda}_1', \quad \lambda_2' = -\tfrac{v_L}{v_D^d}\hat{Y}_b^d - \tfrac{v_d}{v_D^d}\hat{\lambda}_2'. \end{gathered} \tag{2.21}$$

In what follows, we will discuss the phenomenology of these new superpotential written in unhatted fields.

2.4 Fermion mass matrices and diagonalization

We assume that VEVs take the form eq. (2.12) and (2.17), from which we obtain the fermion mass matrices.

2.4.1 Quark sector

The quark mass matrices are given by

$$\mathbf{m}^u = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}Y_c^u v_3^u e^{-i\theta_3^u} & Y_b^u v_D^u e^{-i\theta^u} \\ \sqrt{2}Y_c^u v_3^u e^{-i\theta_3^u} & 0 & Y_b^u v_D^u e^{-i\theta^u} \\ -Y_{b'}^u v_D^u e^{-i\theta^u} & Y_{b'}^u v_D^u e^{-i\theta^u} & \sqrt{2}Y_a^u v_3^u e^{-i\theta_3^u} \end{pmatrix}, \tag{2.22}$$

$$\mathbf{m}^d = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}Y_c^d v_3^d e^{-i\theta_3^d} & Y_b^d v_D^d e^{-i\theta^d} \\ \sqrt{2}Y_c^d v_3^d e^{-i\theta_3^d} & 0 & Y_b^d v_D^d e^{-i\theta^d} \\ -Y_{b'}^d v_D^d e^{-i\theta^d} & Y_{b'}^d v_D^d e^{-i\theta^d} & \sqrt{2}Y_a^d v_3^d e^{-i\theta_3^d} \end{pmatrix}. \tag{2.23}$$

We can bring the mass matrices above to the form

$$\hat{\mathbf{m}}^u = m_t\begin{pmatrix} 0 & q_u/y_u & 0 \\ -q_u/y_u & 0 & b_u \\ 0 & b_u' & y_u^2 \end{pmatrix}, \tag{2.24}$$

[3] In Ref [22], possibilities of the alignment have been discussed in the framework of U(1) horizontal symmetries.

by $\pi/4$ rotations on the Q_6 doublet fermions and appropriate phase rotations. This mass matrix can be then diagonalized by orthogonal matrices $O^u_{L,R}$ as[4]

$$O^{uT}_L \hat{\mathbf{m}}^u O^u_R = \begin{pmatrix} m_u & 0 & \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \tag{2.25}$$

and similarly for $\mathbf{m}^d$.

For the set of the parameters

$$\begin{aligned} &\theta_q = \theta^d_3 - \theta^d - \theta^u_3 + \theta^u = -1.25, q_u = 0.0002150, b_u = 0.04440, b'_u = 0.09300, \\ &y_u = 0.99741, q_d = 0.005040, b_d = 0.02500, b'_d = 0.7781, y_d = 0.7970, \end{aligned} \tag{2.26}$$

we obtain

$$\begin{aligned} m_u/m_t &= 1.11 \times 10^{-5}, \qquad m_c/m_t = 4.14 \times 10^{-3}, \\ m_d/m_b &= 1.22 \times 10^{-3}, \qquad m_s/m_b = 2.07 \times 10^{-2}, \\ |V_{\rm CKM}| &= \begin{pmatrix} 0.97440 & 0.2267 & 0.00388 \\ 0.2265 & 0.9731 & 0.0421 \\ 0.00924 & 0.0412 & 0.9991 \end{pmatrix}, \quad \sin 2\beta(\phi_1) = 0.723. \end{aligned} \tag{2.27}$$

These values are consistent with present experimental values [8]. So, we see that the model can well reproduce the experimentally measured parameters. Moreover, since the CKM parameters and the quark masses are related to each other because of the family symmetry, we find that *nine* independent parameters (eq. (2.26)) of the model can well describe *ten* physical observables: there is *one* prediction. An example of the prediction is $|V_{td}/V_{ts}|$, whose experimental value has been obtained from the measurement of the mass difference Δm_{B_s} of the B^0_s meson [9]:

$$\begin{aligned} \text{Model} : |V_{td}/V_{ts}| &= 0.21 - 0.23, \\ \text{Exp.} : |V_{td}/V_{ts}| &= 0.208 \, {}^{+0.001}_{-0.002} \, (\text{exp.}) \, {}^{+0.008}_{-0.006} \, (\text{theo.}). \end{aligned} \tag{2.28}$$

Finally, we give the unitary matrices that rotate the quarks for the choice of the parameters given in eq. (2.26):

$$\begin{aligned} U_{uL} = &\begin{pmatrix} 0.706 & 0.0366 & 1.42 \times 10^{-5} \\ -0.706 & -0.0366 & -1.42 \times 10^{-5} \\ 0 & 0 & 0 \end{pmatrix} \\ &+ e^{2i\Delta\theta^u} \begin{pmatrix} 0.0366 & -0.705 & 0.0313 \\ 0.0366 & -0.705 & 0.0313 \\ -0.00231 \, e^{-i\Delta\theta^u} & 0.0441 \, e^{-i\Delta\theta^u} & 0.999 \, e^{-i\Delta\theta^u} \end{pmatrix}, \end{aligned} \tag{2.29}$$

[4]The form of the mass matrix is known as the next-neighbor interaction form [6, 7].

$$U_{dL} = \begin{pmatrix} 0.695 & 0.130 & 0.00345 \\ -0.695 & -0.130 & -0.00345 \\ 0 & 0 & 0 \end{pmatrix}$$
$$+e^{2i\Delta\theta^d} \begin{pmatrix} 0.130 & -0.695 & 0.0111 \\ 0.130 & -0.695 & 0.0111 \\ -0.00769\ e^{-i\Delta\theta^d} & 0.0146\ e^{-i\Delta\theta^d} & 1.00\ e^{-i\Delta\theta^d} \end{pmatrix}, \tag{2.30}$$

$$U_{uR} = e^{i\theta_3^u} \begin{pmatrix} 0.0366 & 0.703 & 0.0657 \\ 0.0366 & 0.703 & 0.0657 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.31}$$
$$+e^{2i\Delta\theta^u + i\theta_3^u} \begin{pmatrix} -0.706 & 0.0367 & -6.73 \times 10^{-6} \\ 0.706 & -0.0367 & 6.74 \times 10^{-6} \\ -0.00484\ e^{-i\Delta\theta^u} & -0.0928\ e^{-i\Delta\theta^u} & 0.996\ e^{-i\Delta\theta^u} \end{pmatrix},$$

$$U_{dR} = e^{i\theta_3^d} \begin{pmatrix} 0.134 & 0.427 & 0.548 \\ 0.134 & 0.427 & 0.548 \\ 0 & 0 & 0 \end{pmatrix}$$
$$+e^{2i\Delta\theta^d + i\theta_3^d} \begin{pmatrix} -0.675 & 0.212 & -7.01 \times 10^{-5} \\ 0.675 & -0.212 & 7.01 \times 10^{-5} \\ -0.232\ e^{-i\Delta\theta^d} & -0.739\ e^{-i\Delta\theta^d} & 0.633\ e^{-i\Delta\theta^d} \end{pmatrix}. \tag{2.32}$$

The unitary matrices above will be used when discussing R-parity violating processes in section 3.

2.4.2 Lepton sector

The mass matrix in the charged lepton sector is:

$$\mathbf{m}^e = \frac{1}{2} \begin{pmatrix} -Y_c^e & Y_c^e & Y_b^e \\ Y_c^e & Y_c^e & Y_b^e \\ Y_{b'}^e & Y_{b'}^e & 0 \end{pmatrix} v_D^d e^{-i\theta^d}. \tag{2.33}$$

It is diagonalized by the biunitary transformation:

$$U_{eL}^\dagger \mathbf{m}^e U_{eR} = \begin{pmatrix} m_e & 0 & \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{2.34}$$

One finds that U_{eL} and U_{eR} can be approximately written as

$$U_{eL} = \begin{pmatrix} \epsilon_e(1-\epsilon_\mu^2) & (1/\sqrt{2})(1-\epsilon_e^2+\epsilon_e^2\epsilon_\mu^2) & 1/\sqrt{2} \\ -\epsilon_e(1+\epsilon_\mu^2) & -(1/\sqrt{2})(1-\epsilon_e^2-\epsilon_e^2\epsilon_\mu^2) & 1/\sqrt{2} \\ 1-\epsilon_e^2 & -\sqrt{2}\epsilon_e & \sqrt{2}\epsilon_e\epsilon_\mu^2 \end{pmatrix}, \tag{2.35}$$
$$U_{eR} = \begin{pmatrix} -\epsilon_e^2(1-\epsilon_\mu^2/2) & -1 & 0 \\ 1-\epsilon_\mu^2/2 & -\epsilon_e^2(1-\epsilon_\mu^2) & \epsilon_\mu \\ -\epsilon_\mu & \epsilon_e^2\epsilon_\mu & 1-\epsilon_\mu^2/2 \end{pmatrix} e^{i\theta^d}, \tag{2.36}$$

and small parameters ϵ_e, ϵ_μ are defined as

$$\epsilon_e = \frac{m_e}{\sqrt{2} m_\mu} = 3.42 \times 10^{-3}, \ \epsilon_\mu = \frac{m_\mu}{m_\tau} = 5.94 \times 10^{-2}. \tag{2.37}$$

In the limit $m_e = 0$, the unitary matrix U_{eL} becomes

$$\begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{pmatrix},$$

which is the origin for a maximal mixing of the atmospheric neutrinos.

As for the neutrino sector, we assume that a see-saw mechanism [10] takes place. However, we do not present the details of the neutrino sector here because there is no need to know it in the following analysis. We obtain some specific predictions of our model: (i) only an inverted mass hierarchy $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$ is consistent with the experimental constraint $|\Delta m^2_{21}| < |\Delta m^2_{23}|$, (ii) the $(e, 3)$ element of the MNS matrix is given by $|U_{e3}| \simeq \epsilon_e$. See refs. [2, 3] for details.

2.5 Soft supersymmetry breaking sector

Next we consider the soft SUSY breaking sector, which respects the family symmetry. If three generations of a family is put into a one-dimensional and two-dimensional irreps of any dihedral group, then the soft scalar mass matrix for sfermions has always a diagonal form:

$$\mathbf{m^2}_{(\tilde{q},\tilde{\ell})LL} = \begin{pmatrix} m^2_{(\tilde{q},\tilde{\ell})1L} & 0 & 0 \\ 0 & m^2_{(\tilde{q},\tilde{\ell})1L} & 0 \\ 0 & 0 & m^2_{(\tilde{q},\tilde{\ell})3L} \end{pmatrix}, \tag{2.38}$$

$$\mathbf{m^2}_{\tilde{a}RR} = \begin{pmatrix} m^2_{\tilde{a}1R} & 0 & 0 \\ 0 & m^2_{\tilde{a}1R} & 0 \\ 0 & 0 & m^2_{\tilde{a}3R} \end{pmatrix} \qquad (a = u, d, e).$$

Note that the mass of the first two generations are degenerated because of the family symmetry.[5] Further, since the trilinear interactions (A-terms) are also Q_6 invariant, the left-right mass matrices have the form

$$\left(\mathbf{m}^2_{\tilde{a}LR}\right)_{ij} = A^a_{ij} \left(\mathbf{m}^a\right)_{ij} \quad (a = u, d, e), \tag{2.39}$$

where A^a_i's are free parameters of dimension one, and the fermion masses $\mathbf{m}$'s are given in eq. (2.22), (2.23) and (2.33). They are real, because we impose CP invariance at the Lagrangian level.

We approximate that the squark and slepton masses are given only by eq. (2.38), that is, trilinear terms eq. (2.39) are negligible [2].

[5] SUSY flavor and CP problem can be avoided thanks to such partially degenerated (eq. (2.38)) and aligned (eq. (2.39)) soft terms because of the family symmetry [1, 2].

3. R-parity violation

Since Q_6 family symmetry controls the whole flavor structure of the model, the form of the R-parity violating couplings are also constrained by the family symmetry. We find that only possible trilinear couplings allowed by the symmetry can be written as

$$W_{\not R} = \lambda L_3 L_I E_I^c + \lambda_1' L_I (i\sigma^2)_{IJ} Q_3 D_J^c + \lambda_2' L_I (\sigma^1)_{IJ} Q_J D_3^c \tag{3.1}$$

in the physical lepton doublet L_I defined in eq. (2.16) with the coupling constants in eq. (2.21). Here the superpotential $W_{\not R}$ is written in the flavor eigenstates, so the mixing matrices eqs. (2.29)$\sim$(2.32), (2.35) and (2.36) should appear when we rotate the fermion components into their mass eigenstates. On the other hand, these matrices do not appear from sfermion components, because we approximate that sfermions are in the mass eigenstate basis. In the present model, there are only three R-parity violating trilinear interactions allowed by the family symmetry, and baryon number violating terms $\lambda''_{ijk} U_i^c D_j^c D_k^c$ are forbidden by the symmetry. It should be compared with the MSSM case in which there are 45 trilinear couplings. These interactions can generate a lot of new processes which have not been observed yet such as lepton flavor violating (LFV) processes, or new contributions to already observed processes. Many authors have studied phenomenology of R-parity violation and obtained constraints on each coupling constant corresponding to each process in the MSSM case.[6]

In this section, we obtain constraints on the coupling constants $\lambda, \lambda'_{1,2}$ at the weak scale. Since the various new processes generated by the interactions $W_{\not R}$ depend only on the three coupling constants, we can predict ratios of new processes independent of λs. We will also find the ratios of the LFV processes in this section. As mentioned before, we assume that the R-parity violating couplings λ and $\lambda'_{1,2}$ are real and positive.[7]

3.1 Constraint on λ

In this subsection, we consider the constraint on $\lambda L_3 L_I E_I^c$ operator. The most stringent constraint on λ is obtained from both $\mu \to eee$ and neutrinoless double beta decay, both the processes give similar bound.

First we show the bound on λ from $\mu \to eee$ process [14−17, 35]. The decay process $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ is generated by tree-level t- and u- channel sneutrino exchange (figure 1), and its effective Lagrangian is given by

$$\mathcal{L}_{eff} = \lambda^2 A_L \left(\bar{\ell}_i P_L \ell_m\right)\left(\bar{\ell}_j P_R \ell_k\right) + \lambda^2 A_R \left(\bar{\ell}_i P_R \ell_m\right)\left(\bar{\ell}_j P_L \ell_k\right) + (i \leftrightarrow j), \tag{3.2}$$

where the coefficients are

$$A_L = \frac{1}{m_{\tilde{\ell}1L}^2}(U_{eR}^\dagger)_{iJ}(U_{eR})_{Jk}(U_{eL}^\dagger)_{j3}(U_{eL})_{3m} + \frac{1}{m_{\tilde{\ell}3L}^2}(U_{eR}^\dagger)_{iJ}(U_{eL})_{Jm}(U_{eL}^\dagger)_{jK}(U_{eR})_{Kk}, \tag{3.3}$$

$$A_R = \frac{1}{m_{\tilde{\ell}1L}^2}(U_{eR}^\dagger)_{jJ}(U_{eR})_{Jm}(U_{eL}^\dagger)_{i3}(U_{eL})_{3k} + \frac{1}{m_{\tilde{\ell}3L}^2}(U_{eR}^\dagger)_{jJ}(U_{eL})_{Jk}(U_{eL}^\dagger)_{iK}(U_{eR})_{Km}, \tag{3.4}$$

[6] See ref. [12, 38] and references therein.

[7] CP violation induced by the R-parity violating trilinear couplings in the soft SUSY breaking sector has been studied in ref. [39].

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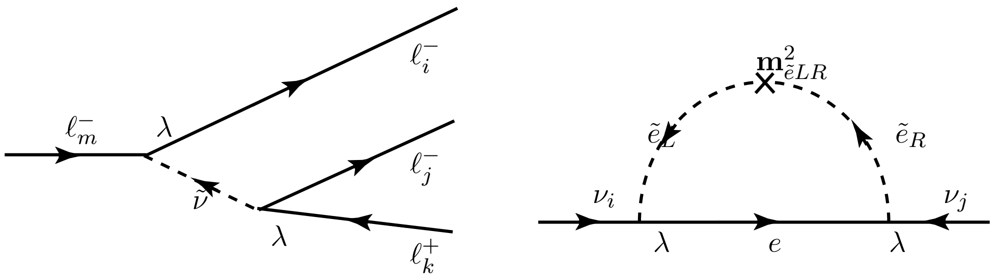

Figure 1: R-parity violating contributions to the decay processes $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ (left) and the neutrino mass (right).

with mixing matrices in eq. (2.35), (2.36). In our approximation, sneutrino mass is the same as that of the left-handed slepton defined in eq. (2.38). From this Lagrangian, the branching ratio of $\mu \to eee$ is given by,

$$BR(\mu \to eee) = \frac{4\lambda^4}{64G_F^2}\left[|A_L|^2 + |A_R|^2\right] BR(\mu \to e\bar\nu_e\nu_\mu). \tag{3.5}$$

The requirement that this branching ratio should not exceed the experimental bound $BR(\mu \to eee)^{\rm exp} < 1.0 \times 10^{-12}$ provides a constraint on λ

$$\lambda < 1.4 \times 10^{-2}\left(\frac{m_{\tilde\ell L}}{100{\rm GeV}}\right), \tag{3.6}$$

where we have assumed $m_{\tilde\ell 1L} = m_{\tilde\ell 3L} \equiv m_{\tilde\ell L}$ in order to forbid the contribution to FCNC processes from the soft scalar mass terms.

Next we show the constraint from neutrinoless double beta decay. The coupling λ can induce radiative neutrino mass at the one loop level shown in figure 1 [14, 20]:

$$\begin{aligned}(\mathcal{M}_\nu)_{ij} = {}& -2\frac{\lambda^2}{(4\pi)^2} m_{ek}\left[(U_{eL})_{3j}(U_{eL})_{Jk} - (U_{eL})_{3k}(U_{eL})_{Jj}\right] \times \\ & \times\Big[(U^\dagger_{eR})_{kI}(U_{eL})_{3i}(\mathbf{m}^2_{\tilde e LR})_{IJ} F(m^2_{ek}, m^2_{\tilde e 1R}, m^2_{\tilde\ell 1L}) \\ & \quad -(U^\dagger_{eR})_{kK}(U_{eL})_{Ki}(\mathbf{m}^2_{\tilde e LR})_{3J} F(m^2_{ek}, m^2_{\tilde e 1R}, m^2_{\tilde\ell 3L})\Big] + (i \leftrightarrow j),\end{aligned} \tag{3.7}$$

where the loop function F is

$$\begin{aligned} F(x,y,z) &= \frac{x\ln x}{(x-y)(x-z)} + \frac{y\ln y}{(y-z)(y-x)} + \frac{z\ln z}{(z-x)(z-y)} \\ &\to \frac{\ln(y/z)}{y-z}, \quad \text{as} \quad x \to 0. \end{aligned} \tag{3.8}$$

We assume that the trilinear terms in the soft SUSY breaking sector are completely aligned with the Yukawa matrices, that is,

$$\mathbf{m}^2_{\tilde e LR} \simeq \mathbf{m}^e \tilde A_e, \tag{3.9}$$

in which $\tilde A_e$ stands for the sum of the A-terms in eq. (2.39) and μ-terms which are not explicitly shown, and $\mathbf{m}^e$ is the mass matrix of charged leptons eq. (2.33). The requirement that the effective neutrino mass $(\mathcal{M}_\nu)_{ee}$ does not exceed a constraint from neutrinoless double beta decay provides a bound on λ:

$$\lambda < 1.1 \times 10^{-2} \left(\frac{(\mathcal{M}_\nu)_{ee}^{\rm exp}}{0.35{\rm eV}}\right)^{1/2} \left(\frac{100{\rm GeV}}{\tilde{M}}\right)^{1/2}, \quad \tilde{M} = \tilde{A}_e \frac{(100{\rm GeV})^2}{m_{\tilde{e}R}^2 - m_{\tilde{\ell}L}^2} \ln \frac{m_{\tilde{e}R}^2}{m_{\tilde{\ell}L}^2}. \tag{3.10}$$

This is the most stringent constraint on λ in the present model: other processes give weaker bounds. For example, a constraint from $\mu \to e\gamma$ is [19]

$$\lambda < 0.22, \tag{3.11}$$

when slepton masses are 100GeV.

3.2 Constraints on λ'_1 and λ'_2

Constraints on $\lambda'_{1,2}$ are obtained from neutral meson mixings [24−29, 35], and on the products $\lambda\lambda'_1$ and $\lambda\lambda'_2$ from leptonic decays of neutral mesons [26, 30−32, 35]. Since both processes are generated at tree level, these give the most stringent bounds on $\lambda'_{1,2}$. Although $\mu - e$ conversion in nuclei [17, 18] is also generated at tree level by $\lambda'_{1,2}$, bounds from this process are weaker than those from neutral meson system. Therefore, we show the calculations only of the neutral meson system.

The neutral meson mixing is generated at the tree level through the exchange of a sneutrino in both s- and t- channels (figure 2). For $K^0 - \bar{K}^0$ mixing, the effective Hamiltonian is obtained as

$$\mathcal{H}_{eff} = \frac{\Lambda'_{I21}\Lambda'^*_{I12}}{m_{\tilde{\ell}1L}^2} (\bar{d}_R s_L)(\bar{d}_L s_R), \tag{3.12}$$

where

$$\Lambda'_{Ijk} = \lambda'_1 (U_{dR}^\dagger)_{jJ} (U_{dL})_{3k} (i\sigma^2)_{IJ} + \lambda'_2 (U_{dR}^\dagger)_{j3} (U_{dL})_{Jk} (\sigma^1)_{IJ}. \tag{3.13}$$

We require that these additional contributions to the mass difference of neutral K meson are smaller than its experimental value:

$$\Delta m_{K^0}^{\not R} = \frac{|\Lambda'_{I21}\Lambda'^*_{I12}|}{2m_{\tilde{\ell}1L}^2} S_{K^0} m_{K^0} f_K^2 B_4(m_{K^0}) < \Delta m_{K^0}^{\rm exp}, \tag{3.14}$$

where m_{K^0} and f_K denote mass and the decay constant of the neutral K meson, and

$$S_{K^0} = \left(\frac{m_{K0}}{m_s(\mu)+m_d(\mu)}\right)^2, \qquad B_4(m_{K^0}) = 1.03, \tag{3.15}$$
$$\Delta m_{K^0}^{\rm exp} = (0.5292 \pm 0.0009) \times 10^{-2} {\rm ps}^{-1}$$

with $\mu = 2\,{\rm GeV}$ [28]. Thus [35],

$$|\Lambda'_{I21}\Lambda'^*_{I12}| < 4.5 \times 10^{-9} \left(\frac{m_{\tilde{\ell}L}}{100{\rm GeV}}\right)^2. \tag{3.16}$$

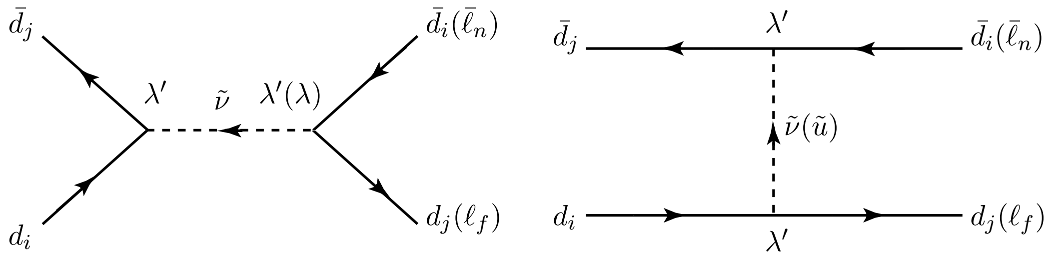

Figure 2: R-parity violating contributions to the neutral meson mixings (or, leptonic dcays of the neutral mesons) in the s- and t- channels. The quantities in parenthesis represent the case of leptonic decays.

The assumptions $\theta_1^d - \theta_3^d = 0$ and $\lambda'_1 = \lambda'_2$ lead to the most stringent constraints on $\lambda'_{1,2}$:

$$\lambda'_1 = \lambda'_2 < 3.1 \times 10^{-3} \left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}} \right). \tag{3.17}$$

Next, we consider the leptonic decays of the neutral K mesons: $K_{L,S} \to e^- e^+, \mu^- \mu^+$ or $e^{\mp}\mu^{\pm}$. At the quark level, these processes are interpreted as a transformation of a down-type quark-antiquark pair (d_i and $\bar{d}_j$) into a charged lepton-antilepton pair (ℓ_f and $\bar{\ell}_n$), which are shown in figure 2. The processes in the s- and t- channels are mediated by sneutrino and u-squark, respectively. In general, the effective Lagrangian for the process $d_i \bar{d}_j \to \ell_f \bar{\ell}_n$ is written as [32]

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= -\frac{1}{2} \mathcal{A}_{ijfn} (\bar{d}_{jR} \gamma^\mu d_{iR})(\bar{\ell}_{fL} \gamma_\mu \ell_{nL}) \\ &+ \frac{1}{2} \frac{m_{di} + m_{dj}}{m_M^2} \left[(\bar{d}_{jL} d_{iR})(\bar{\ell}_{fR} \ell_{nL}) \mathcal{B}_{ijfn} + (\bar{d}_{jR} d_{iL})(\bar{\ell}_{fL} \ell_{nR}) \mathcal{B}^*_{jinf} \right], \end{aligned} \tag{3.18}$$

where each coefficient is defined as

$$\begin{aligned} \mathcal{A}_{ijfn} &= \frac{\lambda_1'^2}{m_{\tilde{q}3L}^2} \left\{ (U_{eL}^\dagger)_{fI} (i\sigma^2)_{IJ} (U_{dR})_{Ji} \right\} \left\{ (U_{eL})_{Ln} (i\sigma^2)_{LK} (U_{dR}^\dagger)_{jK} \right\} \\ &+ \frac{\lambda_2'^2}{m_{\tilde{q}1L}^2} (U_{dR})_{3i} (U_{dR}^\dagger)_{j3} (U_{eL}^\dagger)_{fI} (U_{eL})_{In}, \end{aligned} \tag{3.19}$$

$$\begin{aligned} \mathcal{B}_{ijfn} &= -2 \frac{m_M^2}{m_{di} + m_{dj}} \frac{\lambda}{m_{\tilde{\ell}1L}^2} (U_{eR}^\dagger)_{fK} (U_{eL})_{3n} \times \\ &\times \left[\lambda'_1 (U_{dR})_{Ji} (U_{dL}^\dagger)_{j3} (i\sigma^2)_{KJ} + \lambda'_2 (U_{dR})_{3i} (U_{dL}^\dagger)_{jJ} (\sigma^1)_{KJ} \right], \end{aligned} \tag{3.20}$$

and m_M stands for the mass of the meson composed of quark(d_i)-antiquark($\bar{d}_j$) pair with mass m_{di} and m_{dj}. From the above effective Lagrangian, the decay rate of $d_i \bar{d}_j \to \ell_f \bar{\ell}_n$ is given by

$$\Gamma(d_i \bar{d}_j \to \ell_f \bar{\ell}_n) = \frac{f_M^2}{256 \pi m_M^3} \mathcal{C}_{ijfn} \sqrt{\left\{ m_M^2 - (m_f - m_n)^2 \right\} \left\{ m_M^2 - (m_f + m_n)^2 \right\}}, \tag{3.21}$$

where

$$\mathcal{C}_{ijfn} = \left(m_M^2 - m_f^2\right)|\mathcal{A}_{ijfn}m_f + \mathcal{B}_{ijfn}|^2 + \left(m_M^2 - m_n^2\right)|\mathcal{A}_{ijfn}m_n + \mathcal{B}^*_{jinf}|^2$$
$$-|\mathcal{B}_{ijfn}m_n - \mathcal{B}^*_{jinf}m_f|^2 + m_f m_n\Big[|\mathcal{B}_{ijfn} + \mathcal{B}^*_{jinf}|^2 - |\mathcal{A}_{ijfn}m_n - \mathcal{B}_{ijfn}|^2$$
$$- |\mathcal{A}_{ijfn}m_f - \mathcal{B}^*_{jinf}|^2 + |(m_f+m_n)\mathcal{A}_{ijfn}|^2\Big], \quad (3.22)$$

and f_M is the decay constant of the meson under consideration and $m_{f,n}$ are the corresponding charged lepton masses. In the case of K_L decay, $\mathcal{A}_{ijfn}$ should be replaced to $(\mathcal{A}_{ijfn} - \mathcal{A}_{jifn})/\sqrt{2}$ with $i = 1, j = 2$, and similar for $\mathcal{B}_{ijfn}$ and $\mathcal{B}^*_{jinf}$.

We require that branching ratios should not exceed the experimental bound $BR(K_L \to \mu^\mp e^\pm)^{\rm exp} < 4.7 \times 10^{-12}$ and the experimantal value $BR(K_L \to e^- e^+)^{\rm exp} = 9 \times 10^{-12}$, therefore we obtain

$$\lambda\lambda'_1 < 5.4 \times 10^{-7}\left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}}\right)^2 \quad (3.23)$$

from $K_L \to \mu^\mp e^\pm$, and

$$\lambda\lambda'_2 < 1.1 \times 10^{-8}\left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}}\right)^2 \quad (3.24)$$

from $K_L \to e^- e^+$. The contributions including up type squarks, that is, $\mathcal{A}_{ijfn}$, are negligible in $K_L \to \mu^\mp e^\pm$ because these terms only provide weaker constraints, and vanish in $K_L \to e^- e^+$ because of the replacement mentioned above.

For comparison, bounds on $\lambda\lambda'_{1,2}$ from $\mu - e$ conversion in nuclei are $\lambda\lambda'_1 < 10^{-6}$ and $\lambda\lambda'_2 < 10^{-7}$, which are weaker than eqs. (3.23) and (3.24).

3.3 Predictions for Lepton Flavor Violating processes

Although many processes can be generated by the R-parity violating interactions, we focus on the LFV decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$, where $m = \mu$ or τ, in this subsection. As mentioned in the subsection 3.1, the operator $\lambda L_3 L_I E_I^c$ generates the decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ at tree level when $\lambda \neq 0$. The other two operators in eq. (3.1), $\lambda'_{1,2} LQD^c$, also generate the similar decay processes at one loop level through photon penguin diagrams shown in figure 3, but we found that the bounds on $\lambda'_{1,2}$ are stronger than that on λ in the previous subsections. So we neglect contributions from $\lambda'_{1,2}$ operators to the decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$. Moreover, flavor changing Z boson decay $Z \to \ell_i^- \ell_j^+$ induced by the R-parity violating bilinear terms can contribute to $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ processes. Since branching ratios of these decays are propotional to $\sin^2\xi$, their effects are also negligible [23]. Besides these R-parity violating contributions, there are two other contributions to these processes by Higgs bosons. Since the charged leptons couple to the neutral Higgs bosons, these Yukawa interactions generate LFV processes at one loop level [40]. However, these effects are enhanced only when $\tan\beta$ is large. So, we assume that these are negligible because $\tan\beta$ is small enough. Moreover, since there are three generations of both up and down type Higgs doublet in this model, LFV processes mediated by the neutral Higgs bosons are generated at tree level. However, the branching ratio of the $\mu \to eee$ from these effects is $BR \sim 10^{-16}$ because of the smallness of the Yukawa couplings when the neutral Higgs boson mass is 100GeV. So, these contributions can also be negligible compared to those from λLLE^c couplings unless $\lambda < 10^{-3}$. Therefore, we can approximate that $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ processes are induced at tree level only by λ. In this approximation, the ratios of these processes are independent of λ, but depend on the mixing matrices $U_{eL(R)}$ which reflect the flavor structure of the model. Therefore we find some predictions of LFV decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ in our model.

From the branching ratio eq. (3.5), we can easily find the ratios of processes in the approximation that all scalar masses are equal:[8]

$$\frac{BR(\tau \to eee)}{BR(\tau \to \mu\mu\mu)} \simeq \frac{4\epsilon_\mu^2}{1+\epsilon_\mu^2} = 0.014, \tag{3.25}$$

$$\frac{BR(\tau \to \mu\mu e)}{BR(\tau \to \mu\mu\mu)} \simeq \frac{1-\epsilon_\mu^2+2\epsilon_e^2}{1+\epsilon_\mu^2-2\epsilon_e^2} = 0.99, \tag{3.26}$$

$$\frac{BR(\mu \to eee)}{BR(\tau \to eee)} \simeq \frac{\tau_\mu}{\tau_\tau}\epsilon_\mu^5 \frac{\epsilon_e^2}{2\epsilon_\mu^2+\epsilon_e^2} = 0.0093, \tag{3.27}$$

where small parameters $\epsilon_{e,\mu}$ are given in eq. (2.37) and $\tau_\mu(\tau_\tau)$ stand for the lifetime of the $\mu(\tau)$ lepton. Also, $BR(\tau \to \mu\mu e)$ means $BR(\tau^- \to \mu^-\mu^-e^+)$, and similar for the other processes. One can obtain the ratios of other combinations from the branching ratios listed below:

$$BR(\mu \to eee) \propto \tau_\mu \epsilon_e^2 \left[2(1-2\epsilon_\mu^2)m_{\tilde{\ell}1L}^{-4} + \frac{1}{2}m_{\tilde{\ell}3L}^{-4} - 2(1-\epsilon_\mu^2)m_{\tilde{\ell}1L}^{-2}m_{\tilde{\ell}3L}^{-2}\right], \tag{3.28}$$

$$BR(\tau \to eee) \propto \tau_\tau \left[\epsilon_\mu^2\left(1-\epsilon_\mu^2-4\epsilon_e^2\right)m_{\tilde{\ell}1L}^{-4} + \frac{1}{2}\epsilon_e^2 m_{\tilde{\ell}3L}^{-4}\right], \tag{3.29}$$

$$BR(\tau \to \mu\mu\mu) \propto \tau_\tau \left[\frac{1}{4}\left(1+\epsilon_\mu^2-2\epsilon_e^2-4\epsilon_e^2\epsilon_\mu^2\right)m_{\tilde{\ell}3L}^{-4} - 2\epsilon_e^2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-2}m_{\tilde{\ell}3L}^{-2}\right], \tag{3.30}$$

$$BR(\tau \to ee\mu) \propto \tau_\tau \epsilon_e^2 \left[2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-4} + (\frac{1}{2}-\epsilon_\mu^2)m_{\tilde{\ell}3L}^{-4} - 2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-2}m_{\tilde{\ell}3L}^{-2}\right], \tag{3.31}$$

$$BR(\tau \to \mu\mu e) \propto \tau_\tau \frac{1}{4}\left(1-\epsilon_\mu^2+2\epsilon_e^2-4\epsilon_e^2\epsilon_\mu^2+\frac{1}{4}\epsilon_\mu^4\right)m_{\tilde{\ell}3L}^{-4}, \tag{3.32}$$

$$BR(\tau \to \mu ee) \propto \tau_\tau \left[\epsilon_e^2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-4} + \frac{1}{8}\left(1-2\epsilon_\mu^2+6\epsilon_e^2\epsilon_\mu^2+\frac{3}{2}\epsilon_\mu^4\right)m_{\tilde{\ell}3L}^{-4}\right], \tag{3.33}$$

$$BR(\tau \to \mu e\mu) \propto \tau_\tau \left[\frac{1}{8}\left(1-4\epsilon_e^2+6\epsilon_e^2\epsilon_\mu^2-\frac{3}{4}\epsilon_\mu^4\right)m_{\tilde{\ell}3L}^{-4} + 2\epsilon_e^2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-2}m_{\tilde{\ell}3L}^{-2}\right], \tag{3.34}$$

where the common factor is not shown explicitly.

4. Conclusion

We have considered the properties of R-parity violating operators in a SUSY model with non-Abelian discrete Q_6 family symmetry. The family symmetry can reduce the number

[8]From the conditions to suppress $\mu \to e + \gamma$ process from the scalar mass terms, slepton masses are required to be degenerated with mass differences of order 10^{-1} [4, 2].

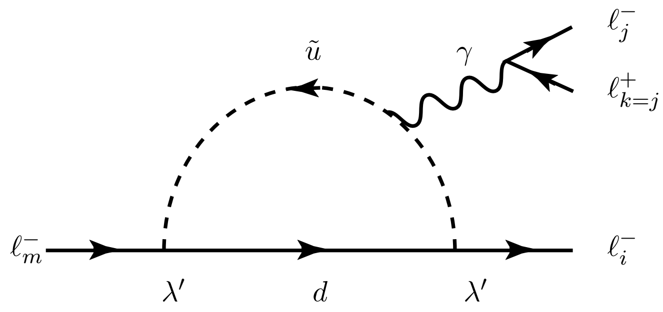

Figure 3: The photon penguin contributions to the decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_{k=j}^+$. These contributions can be negligible compared to the tree level processes induced by the coupling λ.

of parameters in the Yukawa sector, and explain the fermion masses and mixings between generations. It can also reduce the number of R-parity violating couplings and determine the form of those. Only three trilinear couplings are allowed, and the baryon number violating operators are forbidden by the symmetry in our model. We derived upper bounds on these couplings: $\lambda < O(10^{-2})$, $\lambda'_{1,2} < O(10^{-4})$, and obtained the predictions on the ratios of the LFV decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ which do not depend unknown parameters. The results reflect the properties of the family symmetry because these predictions contain the mixing matrices of the charged lepton sector which is written by masses of the charged leptons. Our predictions can be testable at future experiments because the superB factory [41] or LHC [42] will have the sensitivity $BR \sim 10^{-(8\div 9)}$ for LFV τ decays.

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